Improving QCD with fermions: the 2 dimensional case

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We study QCD in 2 dimensions using the improved lattice fermionic Hamiltonian proposed by Luo, Chen, Xu and Jiang. The vector mass and the chiral condensate are computed for various $SU(N_C)$ gauge groups. We do observe considerable improvement in comparison with the Wilson quark case.

1. INTRODUCTION

Wilson's lattice quark formulation has $O(a)$ errors, inducing systematic uncertainties when extracting continuum physics. The most efficient way for reducing these errors is the Symanzik improvement. There have been several reasonable proposals:
(a) Hamber and Wu proposed the first improved action [1] to remove the $O(a)$ error.
(b) Correspondingly, Luo, Chen, Xu, and Jiang constructed an improved Hamiltonian [2].
(c) For other proposals, see Refs. [3].

There have been troubles in the Lagrangian (action) formulation: i.e., it is extremely difficult to study S-matrix and cross sections, wave functions of vacuum, hadrons and glueballs, QCD at finite baryon density, or the computation of QCD structure functions in the region of small x_B and Q^2.

The Hamiltonian approach is a viable alternative [4] and some very interesting results [5–7] have recently been obtained. Workers in Lagrangian formulation nowadays have followed similar ideas by considering anisotropic lattices to improve the spectrum computation.

The purpose of this work is to show that in the case of QCD$_2$, the improved Hamiltonian theory proposed by Luo, Chen, Xu, and Jiang [2] can significantly reduce the $O(a)$ errors.

2. IMPROVED QCD WITH QUARKS

The fermionic Hamiltonian [8] with Wilson quarks with $O(a)$ errors is

$$H_f = m \sum_x \bar{\psi}(x)\psi(x)$$

$$+\frac{1}{2a}\sum_{x,k}\bar{\psi}(x)\gamma_k U(x,k)\psi(x+k)$$

$$+\frac{r}{2a}\sum_{x,k}[\bar{\psi}(x)\psi(x)-\bar{\psi}(x)U(x,k)\psi(x+k)], \quad (1)$$

Luo, Chen, Xu and Jiang's improved Hamiltonian [2] is

$$H_f^{improved} = m\sum_x \bar{\psi}(x)\psi(x) + \frac{r}{2a}\sum_{x,k}\bar{\psi}(x)\psi(x)$$

$$+\frac{b_1}{2a}\sum_{x,k}\bar{\psi}(x)\gamma_k U(x,k)\psi(x+k)$$

$$+\frac{b_2}{2a}\sum_{x,k}\bar{\psi}(x)\gamma_k U(x,2k)\psi(x+2k)$$

$$-c_1\frac{r}{2a}\sum_{x,k}\bar{\psi}(x)U(x,k)\psi(x+k)$$

$$-c_2\frac{r}{2a}\sum_{x,k}\bar{\psi}(x)U(x,2k)\psi(x+2k). \quad (2)$$

Here $U(x,2k)=U(x,k)U(x+k,k)$ and b_1,b_2,c_1 and c_2 are chosen as

$$b_1=\frac{4}{3}, b_2=-\frac{1}{6}, c_1=\frac{4}{3}, c_2=-\frac{1}{3} \quad (3)$$

to classically cancel the $O(a)$ error. These coefficients are the same for any d+1 dimensions and gauge group. In 1+1D, there is only color-electric energy in the gluonic sector, therefore classical improvement is sufficient.

With the absence of the $O(ra)$ errors, we expect that we can extract the continuum physics in a more reliable way.

3. PHYSICAL RESULTS

We have constructed the wave functions of vacuum and quark-antiquark vector state, and discussed the mixing problem of the operator $\langle\bar{\psi}\psi\rangle$ with the identity. For details, see Ref. [9].

Figure 1 plots $-\langle\bar{\psi}\psi\rangle_{sub}/(gN_C)$ as a function of $1/g^2$ in SU(3) gauge theory ($N_C=3$) for Wilson parameter $r=0.1$ (crosses) and $r=1$ (diamonds). Here *sub* means the lattice data subtract the contribution of free fermions. Figure 3 plots aM_V/g as a function of $1/g^2$, with M_V being the vector mass. As one sees, the results for $r=1$ deviate obviously from those for $r=0.1$, which is attributed to the $O(ra)$ error of the Wilson term.

The results from the improved Hamiltonian are presented in Fig. 2, and Fig. 4. One observes that the differences between the results for $r=1$ and $r=0$ are significantly reduced. Most impressively, the data for the quark condensate coincide each other. A similar r test has also been used by workers in Lagrangian formulation for checking the efficiency of the improvement program.

For the interest of workers outside the lattice community [10], we also show the results as a function of $1/N_C^2$ in Fig. 5 and Fig. 6.

It is worth mentioning the improved Hamiltonian formulation for pure gauge theory [11,12] proposed by Luo, Guo, Kröger, and Schütte is also giving promising results.

In conclusion, the results from the improved Hamiltonian [2] have much less systematic errors than those from Wilson's. We believe that its application to QCD in 3+1 dimensions will be very encouraging.

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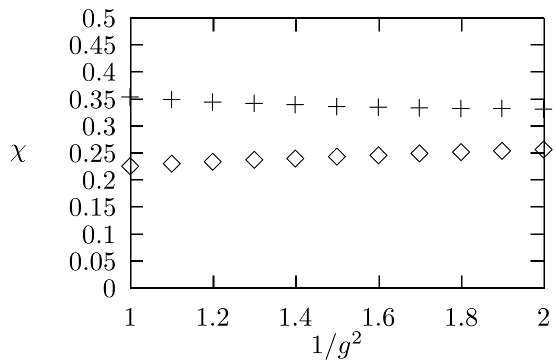

Figure 1. $\chi = -\langle\bar{\psi}\psi\rangle_{sub}/(gN_c)$ versus $1/g^2$ for $N_C = 3$ with Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

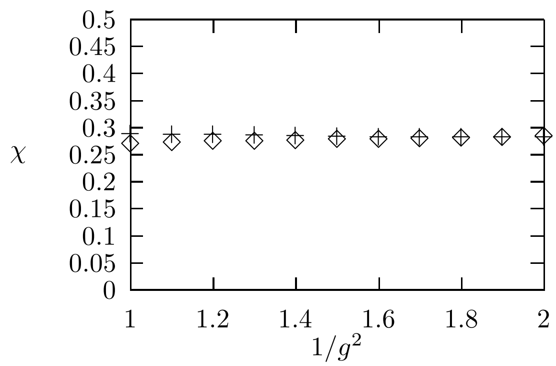

Figure 2. $\chi = -\langle\bar{\psi}\psi\rangle_{sub}/(gN_c)$ versus $1/g^2$ for $N_C = 3$ with improved Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

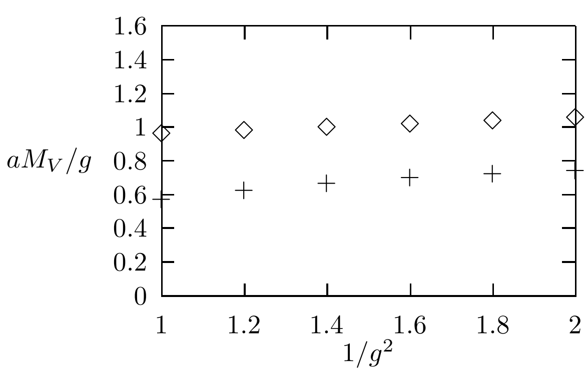

Figure 3. aM_V/g versus $1/g^2$ for $N_C = 3$ with Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

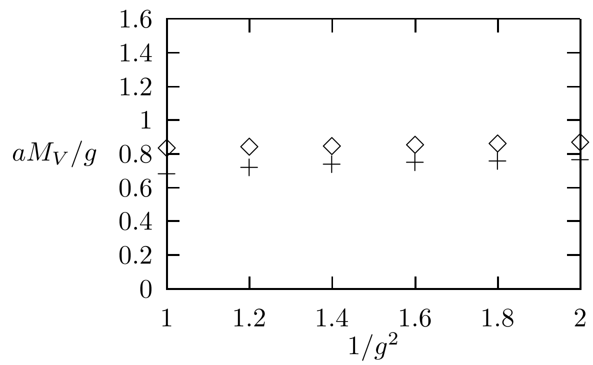

Figure 4. aM_V/g versus $1/g^2$ for $N_C = 3$ with improved Wilson fermions. Crosses: $r = 0.1$, Diamonds: $r = 1$.

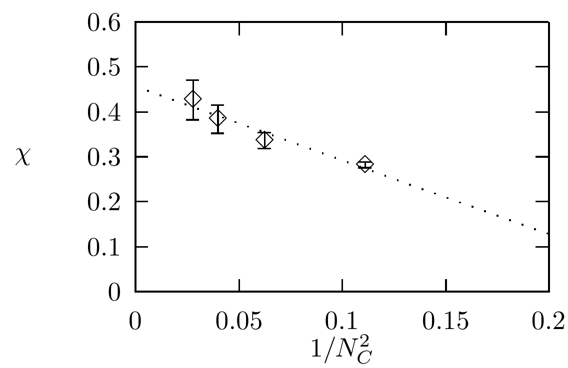

Figure 5. $\chi = -\langle\bar{\psi}\psi\rangle_{cont}/(eN_C)$ versus $1/N_C^2$ in the continuum. The error bars are estimated from the data for different r.

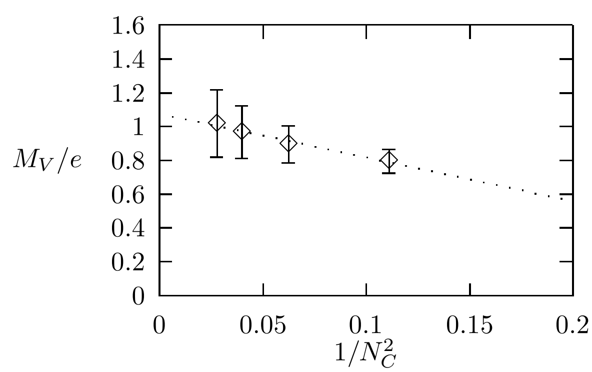

Figure 6. M_V/e versus $1/N_C^2$ in the continuum. The error bars are estimated from the data for different r.